Fonts for Mathematics

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Basic Requirements for a Text Font

To be usable for mathematical typesetting,
a text font should have

- a calm and unobtrusive design,
- clear, differentiated letterforms,
- at least two weights,
  with a difference recognizable in a single letter:

  $A\ \boldsymbol{A}\quad x\ \boldsymbol{x}\quad f\ \boldsymbol{f}\quad \mathrm{R}\ \mathbf{R}$

  A difference visible in text may not be sufficient:

  $\mathbf{text}\quad \mathbf{text}\qquad \mathbf{x}\ \mathbf{x}\qquad \boldsymbol{x}\ \boldsymbol{x}$

Basic Requirements for a Text Font

The most important font in math is Italic.

Italic should

- be really italic (not oblique Roman):

  a a f f g g

- have letterforms clearly distinct from Roman:

  s s x x y y

- have a distinct slant:

  A A X X c c

SansSerif and Math

In general, seriffed text fonts should be used.
SansSerif fonts are not suitable for (complicated) math.

Many letters

- are too similar to certain symbols:

  c ⊂ x × T T

- have too little difference between upper- and lowercase:

  C c O o S s V v X x Z z

Every single letter must be recognizable when appearing
- alone, - in smaller print, or - not on the baseline.

SansSerif and Math

Examples of confusable letters and symbols:

C c C c ⊂ O o O o ○ ∘ 0 U u U u ∪

C c *C* *c* ⊂ O o *O* *o* ○ ∘ 0 U u *U* *u* ∪

I l I l 1 T T ⊤ X x X x χ χ ×

I l *I* *l* 1 T *T* ⊤ X x *X* *x* ×

Confusable Greek letters and symbols:

Δ Δ △ $\triangle$ Λ Λ ∧ $\bigwedge$ ϵ ϵ ∈

Θ θ Θ θ ⊖ ⊖ Π Π ⊓ $\prod$ $\bigsqcap$ ∇ $\bigtriangledown$ ∇

Special Requirements

Most text fonts will fail on one or some of the following more special requirements:

- Optical Sizes
- Special Math Italic
- Complete Set of Greek Letters
- Special Letterforms

Optical Sizes

- Very desirable:

A^{A^A} i_{i_i} x^{x_x} $(a+b)^{(a+b)^{(a+b)}}$ (with 3 sizes)

A^{A^A} i_{i_i} x^{x_x} $(a+b)^{(a+b)^{(a+b)}}$ (with 1 size only)

- Some OpenType fonts offer (typically 4) optical sizes:
  - Caption 6–8 point $[abcxyz)$
  - Regular 9–13 point $[abcxyz)$
  - Subhead 14–24 point $[abcxyz)$
  - Display 25–72 point $[abcxyz)$

. . . but math would need *two sizes* below “Regular”.

Math Italic

It is preferable to use a special, slightly wider Math Italic.

- Better separation of text and math.
- Kerning of Math Italic is different.
  Each letter should be read as a single letter,
  not as part of a word:

  text: $abcdef$ math: $abcdef$

- Math Italic could be 5–10 % wider.
- In some fonts the next greater width can be used.

Greeks

- Mathematics needs a complete set of Greek letters.
- The same 4 designs are needed as for Latin: Roman, Italic, Bold, Bold Italic.
- The full set of variant letters is used:

$$\epsilon \quad \theta \quad \kappa \quad \pi \quad \rho \quad \sigma \quad \phi \qquad \beta$$

$$\varepsilon \quad \vartheta \quad \varkappa \quad \varpi \quad \varrho \quad \varsigma \quad \varphi \qquad (\varbeta)$$

- Some additional derived glyphs are used:

$$\nabla \quad \amalg \quad \mho \qquad \lambda \quad \lambda \qquad \text{ə} \quad \text{ə} \qquad \text{ɪ} \quad \textit{ɪ}$$

Special Letterforms

Some Italic letterforms need special attention:

- the “open g” is preferred: g, not g
- “a” and “alpha”: a α
- “x” and both forms of “kappa”: x κ $\varkappa$
- “y” and “gamma”: y γ
- upright and italic “delta”: δ δ
- “v” and “nu” — ν is not usable: υ ν

  … and w

(“omicron” o and “upsilon” υ are not used)

Alphabetic and Geometric Glyphs

The additional glyphs needed for mathematics could be categorized as:

- "alphabetic" — font-dependent design

  $\infty \quad \nabla \quad \partial \quad \wp \quad \propto \quad \Game \quad * \quad \aleph \quad \sum \quad \int$

- "geometric" or technical — geometric, rather font-independent design

  $+ \quad = \quad \approx \quad < \quad \geq \quad \in \quad \supset \quad \langle \quad / \quad \rangle \quad \times \quad \bigtimes$

- "mixed" — combined design

  $\rightarrow \quad \nearrow \quad \leftrightarrows \quad \{\!| \quad |\!\} \quad \oiint$

Size, Shape, and Weight

All additional glyphs should match the font design in

- Size:

$$a+b \quad g \neq f \quad E \geq F(x)$$
$$a+b \quad g \neq f \quad E \geq F(x)$$

- Shape:

$$x * y \quad i \longrightarrow \infty \quad f \propto g$$
$$x * y \quad i \longrightarrow \infty \quad f \propto g$$

- Weight:

$$f(x) = a + b \quad \boldsymbol{f(x) = a + b} \quad \boldsymbol{f(x) = a + b}$$

Metrics

Glyphs of the same kind should have the same width, as they often appear in (vertical) alignments, e.g.

Relators:	Junctors:	Arrows:
$=$	$+$	$\longrightarrow$
$<$	$-$	$\longleftrightarrow$
$\geq$	$\pm$	$\Longleftarrow$
$\simeq$	$\times$	$\longmapsto$

$$a = b + c$$
$$\leq b + d + e$$

Math Axis

- All formulas are typeset with a horizontal axis.
- The axis' height is font-dependent (standard value: middle height of delimiters).
- Many symbols are vertically centered on this axis:

$$+ - \pm \cdot \times = < \leq > \geqq \lessgtr \equiv \neg \setminus$$

- Fraction bars, delimiters, integrals and big operators also get vertically centered:

$$f(z) = \frac{1}{2\pi \mathrm{i}} \int_{K_1} \left(\sum_{v=0}^{\infty} \frac{f(\zeta)(z-a)^v}{(\zeta-a)^{v+1}} \right) \mathrm{d}\zeta$$

Basic Math Glyphs

Most OpenType fonts contain a few math characters.
These should have the following shapes:

- always upright, never italic / oblique / slanted:

  $+ - \pm \cdot \times \div \neg \quad = \neq \approx \sim < > \leq \geq$

  $\infty \quad | \, ¦ \, \Diamond \, \wedge \quad \prod \sum$ (as “big operators”)

- both upright and italic:

  $\partial \, \mu \, \pi \, \Delta \, \Omega \quad \partial \, \mu \, \pi \, \Delta \, \Omega$

- one form only (either upright or slanted):

  $\int \sqrt{} \quad \int \sqrt{}$

Problematic Glyphs

- "infinity" should be "alphabetic" and at lowercase size:

  too small: $f(x) \to \infty$ good: $f(x) \to \infty$

- upright and italic "partialdiff" must be different:

  too similar: $\partial\, \partial$ good: $\partial\, \partial$

- big operators and integrals are needed at (at least) two sizes:

  too small / one size only: $\sum \prod \int$

  better: $\sum \sum \prod \prod \int \int \int$

Big Operators and Integrals

Big Operators are needed in at least 2 sizes, for inline and display setting.

Integrals should be available in at least 3 sizes.

… in the line $\sum_{k=1}^{3} a_i$ like this and $\int f(x)\,\mathrm{d}x$ (without conditions) and $\int_0^1 f(x)\,\mathrm{d}x$ (with conditions); in display:

$$\sum_{k=1}^{3} a_i = a_1 + a_2 + a_3; \qquad \int_0^1 f(x)\,\mathrm{d}x.$$

Larger and Extensible Glyphs

Delimiters and **Root Symbols** are needed
in larger (and extensible) versions — especially designed.
Simple scaling would lead to too heavy glyphs:

$$\left\{\left\{\left\{\left\{\left\{\left\{\left\{\left\{\left\{a+b\right\}\right\}\right\}\right\}\right\}\right\}\right\}\right\}\right\}$$

$$\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{x^2}}}}}} \qquad \sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{x^2}}}}}} \qquad \sqrt{}$$

Larger and Extensible Glyphs

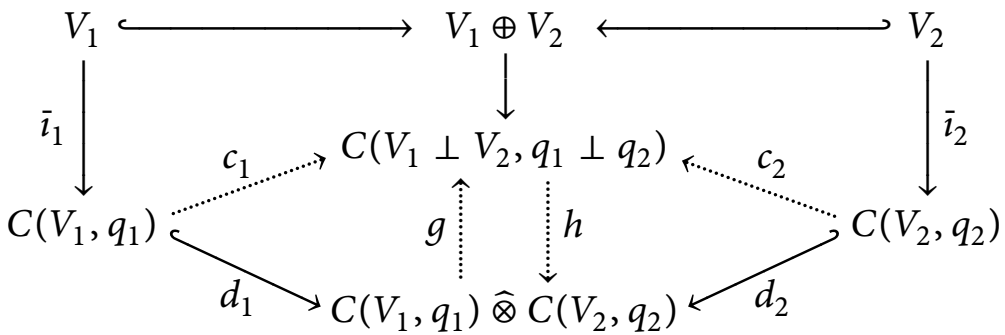
In math, **accents** may be set over several characters at once:

$$\overbrace{opqrs} \quad \widetilde{ABC} \quad \overrightarrow{\mathrm{d}f}$$

Text accents may not be distinct enough:

$$sueño \quad \tilde{n}\,\bar{n} \quad \widetilde{n}\,\overline{n}$$

Extensible **arrows** are needed, too:



Font Usage in Mathematics

Italic is used for

- variables
- functions and operators
- quantities and constants in physics

$$ab = c$$

$$f(x) \quad \zeta(z)$$

$$h \quad \hbar \quad m_{\mathrm{u}} \quad N_{\mathrm{A}}$$

Bold Italic: mainly for vectors and matrices

$$\boldsymbol{x} = (x_1, \ldots, x_n)^{\mathrm{T}}$$

$$\boldsymbol{A} \cdot \boldsymbol{A}^{-1} = \boldsymbol{E}$$

Font Usage in Mathematics

Roman / Upright is used for

- all arithmetical symbols $+ - = \leq \geq \times$
- all delimiters $( ) \{ \} [ ]$
- digits and punctuation $3.1415 \quad 2{,}7182$
- constants, operators, indices, etc. with a fixed meaning $\mathrm{e}^{\pi \mathrm{i}} = -1 \quad F_{\mathrm{eff}}$
- abbreviated function names etc. $\sin x \quad \log 2$
- units in physics $\mathrm{m},\ \mathrm{K},\ \mathrm{kg},\ \mathrm{Hz}$

Boldface: mainly for number sets $\mathbf{N} = \{0, 1, 2, \ldots\}$

Readability of Formulas

The specific tension and the readability of math formulas depend upon this mixture of upright and slanted glyphs:

$$\left(\prod_{j=1}^{n} \hat{x}_j\right) H_c = \frac{1}{2}\hat{k}_{ij} \det \widehat{\boldsymbol{K}}(i \mid i), \qquad i = 1, \ldots, n.$$

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\right) \ln \psi_0(r) = h(r) \qquad 5x_1 + 7x_2 = 6$$

$$(A \Rightarrow B) \wedge A \Rightarrow B \qquad |P_Y| = \sum_{X \geq Y} \mu(Y, X)\, |Q_X|$$

Additional Fonts

All additional fonts should match the base font in size, weight and general style:

- Script
  (mostly uppercase only)
  $\mathscr{A}\ \mathscr{B}\ \mathscr{C}\ \mathscr{D}\ \mathscr{E}\ \mathscr{F}$
- Fraktur
  (upper- and lowercase)
  $\mathfrak{A}\ \mathfrak{B}\ \mathfrak{C}\ \mathfrak{D}\ \mathfrak{a}\ \mathfrak{b}\ \mathfrak{c}$
- Hebrew
  (first 4 letters only)
  ד ג ב א
- Blackboard Bold / Doublestroke
  (see next page)

Blackboard Bold / Doublestroke

- Origin: Representation of boldface in handwriting:

  $\mathbf{R} \rightarrow \mathbb{R} \rightarrow \mathbb{R}$

  $\mathbf{Z} \rightarrow \mathbb{Z} \rightarrow \mathbb{Z}$

- Used mainly for number sets:

  $\mathbb{N}\ \mathbb{Z}\ \mathbb{Q}\ \mathbb{R}\ \mathbb{C}$

- Mostly uppercase letters only, sometimes $\mathbb{1}$ and $\Bbbk$ are used.

- Not an outline font, but a special design — it must look bolder than “Regular”, not lighter

Examples

- Computer Modern

$$\frac{1}{2\pi\mathrm{i}}\int_{\partial G} f(z)\,\mathrm{d}z = \sum_{\nu=1}^{n}(\operatorname{res} f)(z_\nu)$$

$$\bigcup_{i\in I} A_i := \left\{x \,\middle|\, \bigvee_i (i \in I \wedge x \in A_i)\right\}$$

- Euler

$$\frac{1}{2\pi\mathrm{i}}\int_{\partial G} \mathrm{f}(z)\,\mathrm{d}z = \sum_{\nu=1}^{n}(\operatorname{res} \mathrm{f})(z_\nu)$$

$$\bigcup_{\mathrm{i}\in \mathrm{I}} \mathrm{A}_\mathrm{i} := \left\{\mathrm{x} \,\middle|\, \bigvee_\mathrm{i} (\mathrm{i} \in \mathrm{I} \wedge \mathrm{x} \in \mathrm{A}_\mathrm{i})\right\}$$

Examples

- Times & MathTime

$$\frac{1}{2\pi i}\int_{\partial G} f(z)\,dz = \sum_{\nu=1}^{n}(\operatorname{res} f)(z_\nu)$$

$$\bigcup_{i\in I} A_i := \left\{x \;\middle|\; \bigvee_i (i \in I \wedge x \in A_i)\right\}$$

- Lucida

$$\frac{1}{2\pi i}\int_{\partial G} f(z)\,dz = \sum_{\nu=1}^{n}(\operatorname{res} f)(z_\nu)$$

$$\bigcup_{i\in I} A_i := \left\{x \;\middle|\; \bigvee_i (i \in I \wedge x \in A_i)\right\}$$

Examples

- Utopia & Fourier

$$\frac{1}{2\pi \mathrm{i}} \int_{\partial G} f(z)\,\mathrm{d}z = \sum_{\nu=1}^{n} (\operatorname{res} f)(z_\nu)$$

$$\bigcup_{i\in I} A_i := \left\{ x \,\middle|\, \bigvee_i (i \in I \wedge x \in A_i) \right\}$$

- Minion & MnMath

$$\frac{1}{2\pi \mathrm{i}} \int_{\partial G} f(z)\,\mathrm{d}z = \sum_{\nu=1}^{n} (\operatorname{res} f)(z_\nu)$$

$$\bigcup_{i\in I} A_i := \left\{ x \,\middle|\, \bigvee_i (i \in I \wedge x \in A_i) \right\}$$

Conclusion

Major difficulties:

- Design of additional letters and alphabetic glyphs
- Metrics, sidebearings, kerning

Minor difficulties:

- Design of geometric glyphs
- Font dimensions
  (height of math axis, size of geometric glyphs, . . .)
- Technical issues
  (encodings, Unicode, OpenType, . . .)

